expansions for the forces, moments, and transverse forces. Let $\varepsilon_\theta$ denote the circumferential strain. If $\varepsilon_\theta(w_n) \neq 0$, n = 0, 1, then the asymptotic expansions of the moment $M_\theta$ and the transverse forces $N_r$, $N_\theta$ start with the power $\varepsilon^{-2}$.

For $\varepsilon_\theta(w_n) = 0$, n = 0, 1 the plate is inextensible in the circumferential direction.

In conclusion, we note that the asymptotic for the problem of bending a symmetrically assembled anisotropic rectangular laminar shell [5] under strictly nonzero steepness of the family of bonding fibers can be constructed completely analogously (in a formal complication of the computations).

LITERATURE CITED


1. M. I. Vishik and L. A. Lyusternik, "Regular degeneration and the boundary layer for linear differential equations," Usp. Mat. Nauk, 12, No. 5 (1957).
2. S. A. Nazarov, "Ultrapower boundary layer in the problem of bending of a stressed plate," Vestn., Leningrad Univ., Ser. Mat., Mekh., Astr., No. 1 (1980).
3. S. G. Lekhnitskii, Anisotropic Plates [in Russian], OGIZ, Moscow (1947).
4. V. A. Kondrat'ev, "Boundary value problems for elliptic equations in domains with conical or angular points," Trudy, Moskov. Matem. Obshch., 16, 219 (1967).
5. S. A. Ambartsumyan, Theory of Anisotropic Shells [in Russian], Fizmatgiz, Moscow (1961).


# STABILITY OF MAGNETIC SUSPENSION IN A DIRECT-CURRENT MAGNETIC FIELD


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The problem of suspension of a body for a lengthy period of time using permanent magnets has attracted the interest of researchers. A detailed bibliography of studies of this problem, an analysis of the state of the art, and original results have been presented in [1, 2].

The major result achieved has been Ernshaw's theorem, which indicates the instability of such suspension. However this theorem is concerned with steady state situations, and as we will demonstrate below, is inapplicable to dynamic systems.

1. We will consider the configuration of magnets shown in Fig. 1. We will consider motion of an infinitely long rod in the magnetic channel along the axis Ox. The weight of the rod P = mg is compensated by magnets of one sign 1 or 2. Along the channel sides there is a system of permanent magnets of alternating polarity, which interacts with a similar system located on the rod. We will assume that the pole step along the axis Ox is equal to $\lambda = 2\pi/k$, where k is the wave number. We will assume the magnetic material to be saturated with a value of $\mu = 1$ (where $\mu$ is the relative permittivity), as in a vacuum. Considering further that magnet system 3 has a vertical length, we will neglect forces produced by interaction of magnets 3 and 4 during vertical oscillations of the rod.

We will now perform some preliminary calculations. At the point $M_0(x_0, y_0, z_0)$ let there be some magnetic charge q. Its potential at the point M(x, y, z) is equal to $U = q/4\pi\mu_0 r$, $r^2 = (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2$, where $\mu_0$ is the absolute magnetic permittivity of free space. The force produced by interaction of two charges $q^+$, $q^-$ located at these points is given by the expression

$$F = \frac{1}{4\pi\mu_0}\frac{q^+q^-}{r^2}$$

and is directed along the vector joining the charges.

We will consider an infinite magnetic pole located along the axis Oz. For an element of the pole dz the magnetic charge is equal to $dq = \gamma^+dz$, where $\gamma^+$ is the linear charge density. The force of interaction with an analogous elementary charge sectioned from another magnetic


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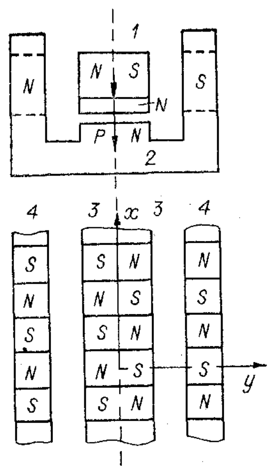

Fig. 1

pole will be:

$$d^2F = \gamma^+\gamma^- dz\, dz_0/(4\pi\mu_0 r^2).$$

We will now define the force with which a charge located in the plane y = δ is attracted to a pole the edge of which lies in the plane y = −δ at coordinate x. Projecting the force $d^2F$ onto the shortest line joining the point $M_0(x_0, y_0 = \delta, z_0)$ to the pole $x' = x$, $y' = -\delta$, $z' = z$, we have

$$d^2R = d^2F \frac{\sqrt{(x-x_0)^2 + 4\delta^2}}{\sqrt{(x-x_0)^2 + 4\delta^2 + (z-z_0)^2}} = \frac{\gamma^+\gamma^-}{4\pi\mu_0} \frac{\sqrt{(x-x_0)^2+4\delta^2}}{\{(x-x_0)^2 + 4\delta^2 + (z-z_0)^2\}^{3/2}}.$$

The force with which an elementary charge is attracted by an infinite pole is given by the integral

$$dR = \frac{\gamma^+\gamma^- dz}{4\pi\mu_0} \int\limits_{-\infty}^{\infty} \frac{\{(x-x_0)^2 + 4\delta^2\}\, dz}{\{(x-x_0)^2 + 4\delta^2 + (z-z_0)^2\}^{3/2}}. \qquad (1.1)$$

Integrating Eq. (1.1) and performing the replacement $z - z_0 = u\sqrt{(x - x_0)^2 + 4\delta^2}$, we obtain [3]

$$dR = \gamma^+\gamma^- dz_0/[2\pi\mu_0\sqrt{(x-x_0)^2 + 4\delta^2}].$$

A pole of unit length along the axis Oz will experience an attractive force

$$R = \frac{\gamma^+\gamma^-}{2\pi\mu_0} \frac{1}{\sqrt{(x-x_0)^2 + 4\delta^2}}. \qquad (1.2)$$

In the case where the charge is distributed along the axis Ox, i.e., $dq = q^- dx$ (where $q^- = d\gamma^-/dx$ is the magnetic charge density) we write Eq. (1.2) in the form

$$d^2R = q^+q^- dx\, dx_0/[2\pi\mu_0\sqrt{(x-x_0)^2 + 4\delta^2}].$$

The projections of this force on the Ox and Oy axes are equal to

$$d^2X = \frac{(x-x_0)\, q^+q^- dx\, dx_0}{2\pi\mu_0 \{(x-x_0)^2 + 4\delta^2\}},$$
$$d^2Y = \frac{2\delta q^+q^- dx\, dx_0}{2\pi\mu_0 \{(x-x_0)^2 + 4\delta^2\}}. \qquad (1.3)$$

First of all, we integrate Eq. (1.3) over the variable x. By doing this we define the force produced upon an elementary magnetic charge by all charges in the plane y = −δ:

$$dX = \frac{q^+(x_0)\,dx_0}{2\pi\mu_0} \int\limits_{-\infty}^{\infty} \frac{(x-x_0)^2 q^-(x)\,dx}{(x-x_0)^2+4\delta^2},$$
$$dY = \frac{q^+(x_0)\,dx_0}{2\pi\mu_0} \int\limits_{-\infty}^{\infty} \frac{2\delta q^-(x)\,dx}{(x-x_0)^2+4\delta^2}. \quad (1.4)$$

Performing the substitution $x - x_0 = u$ in Eq. (1.4) and taking the magnetic charge density in the form

$$q^+(x_0) = q_0 \sin k(x_0+\xi),\ \ q^-(x) = q_0 \sin kx,$$

we find, by symmetry, that

$$dX = \frac{q_0^2}{\pi\mu_0} \sin k(x_0+\xi)\cos kx_0 dx_0 \int\limits_0^{\infty} \frac{u \sin ku\,du}{u^2+4\delta^2},$$
$$dY = \frac{q_0^2}{\pi\mu_0} \sin k(x_0+\xi)\sin kx_0 dx_0 \int\limits_0^{\infty} \frac{2\delta \cos ku\,du}{u^2+4\delta^2}.$$

Since according to [3]

$$\int\limits_0^{\infty} \frac{2\delta\cos ku\,du}{u^2+4\delta^2} = \int\limits_0^{\infty} \frac{u\sin ku\,du}{u^2+4\delta^2} = \frac{\pi}{2}e^{-2\delta k},$$

we have

$$dX = \frac{q_0^2 e^{-2\delta k}}{2\mu_0}\sin k(x_0+\xi)\cos kx_0 dx_0,\quad dY = \frac{q_0^2 e^{-2\delta k}}{2\mu_0}\sin k(x_0+\xi)\sin kx_0 dx_0.$$

From this we define the attractive force per unit surface:

$$X = \frac{q_0^2 e^{-2\delta k}}{2\mu_0}\lim \frac{k}{\pi n}\int\limits_0^{\frac{\pi n}{k}} \sin k(x_0+\xi)\cos kx_0 dx_0 = \frac{q_0^2 e^{-2\delta k}}{2\mu_0}\sin k\xi \lim \frac{k}{\pi n}\int\limits_0^{\frac{\pi n}{k}} \cos^2 kx_0 dx_0,$$

i.e.,

$$X = \frac{q_0^2 e^{-2\delta k}}{2\mu_0}\sin k\xi,\quad Y = \frac{q_0^2 e^{-2\delta k}}{2\mu_0}\cos k\xi. \quad (1.5)$$

We will now consider the force of attraction (or repulsion) between the surface $y = \delta$ and the surface $y = -\delta - l$, formed by the two ends of the magnets. This force can be calculated from Eq. (1.5) by replacing the quantity $2\delta$ by $2\delta + l$ and considering the sign of the force. Then the total force is given by the expression

$$X = \frac{q_0^2 e^{-2\delta k}}{2\mu_0}(1-e^{-lk})^2\sin k\xi,\quad Y = \frac{q_0^2 e^{-2\delta k}}{2\mu_0}(1-e^{-lk})^2\cos k\xi.$$

Considering that the force of interaction between a unit magnetic pole dq and a magnetic field of intensity H is determined by the product $dX = dqH$, from Eq. (1.5) we find the value of the field intensity on the surface itself, at the point corresponding to the maximum

$$H_x = q_0/\mu_0,\quad H_y = q_0/\mu_0.$$

Using the magnetic induction $B = \mu_0 H$, we write the force

$$X = \frac{B^2}{2\mu_0}e^{-2\delta k}(1-e^{-lk})^2\sin k\xi,\quad Y = \frac{B^2}{2\mu_0}e^{-2\delta k}(1-e^{-lk})^2\cos k\xi. \quad (1.6)$$

2. Knowing the force characteristics of the magnetic field of system (1.6), we can construct dynamic equations. We will displace the rod to the right by an amount y. Then the right-hand gap will be $\delta - y$, and the left-hand, $\delta + y$. In this case the attractive (repulsive) force for the right-hand gap will be

$$Y^{+} = -\frac{B^2}{2\mu_0} e^{-2\delta k - yk} (1 - e^{-lk})^2 \cos kx,$$

while for the left-hand gap

$$Y^{-} = \frac{B^2}{2\mu_0} e^{-2\delta k + yk} (1 - e^{-lk})^2 \cos kx.$$

Then the total force will be given by

$$\mathcal{Y} = Y^{+} + Y^{-} = \frac{B^2}{\mu_0} e^{-2\delta k} (1 - e^{-lk})^2 \cos kx \operatorname{sh} ky.$$

Similarly we define the force $\mathcal{X}$ by the expression

$$\mathcal{X} = X^{+} + X^{-} = \frac{B^2}{\mu_0} e^{-2\delta k} (1 - e^{-lk})^2 \sin kx \operatorname{ch} ky. \tag{2.1}$$

Aside from the force $\mathcal{Y}$, there will act a destabilizing force $\mathcal{Y}^*$, produced by the lower magnetic system, the supporting mechanism. For the case where this support system is composed of several magnetic poles, by analogy to Eq. (2.1) we find for the force $\mathcal{Y}^*$

$$\mathcal{Y}^* = \frac{B_1^2}{\mu_0} e^{-2\delta k_1} \sin k_1 y.$$

Here $k_1$ is the wave number of the lower magnetic system. The supporting force will be equal to

$$P = \frac{B_1^2}{\mu_0} e^{-2\delta k_1} \cos k_1 y.$$

At $y = 0$ we have $P = mg$. Hence we find $mg = [\exp(-2\delta k_1)]B_1^2/\mu_0$, then $\mathcal{Y}^* = mg \sin k_1 y$. With consideration of all forces acting we can write the equations of motion of the rod:

$$m\frac{d^2x}{dt^2} = \frac{B^2}{\mu_0} e^{-2\delta k} (1 - e^{-lk})^2 \sin kx \operatorname{ch} ky,$$
$$m\frac{d^2y}{dt^2} = -\frac{B^2}{\mu_0} e^{-2\delta k} (1 - e^{-lk})^2 \cos kx \operatorname{sh} ky + mg \sin k_1 y. \tag{2.2}$$

Introducing the notation

$$\frac{B^2}{\mu_0 m} e^{-2\delta k} (1 - e^{-lk})^2 = A, \quad k_1 = l,$$

we transform Eq. (2.2) to the form

$$d^2x/dt^2 = A \sin kx \operatorname{ch} ky, \quad d^2y/dt^2 = -A \cos kx \operatorname{sh} ky + g \sin ly. \tag{2.3}$$

It can be shown that Eq. (2.3) is the equation of motion

$$d^2x_1/dt^2 = -\partial\Pi(x)/\partial x_1, \quad d^2x_2/dt^2 = -\partial\Pi(x)/\partial x_2$$

in a force field with potential $\Pi(x) = a \operatorname{ch} kx_2 \cos kx_1 + b \cos lx_2$ ($x_1 = x$, $x_2 = y$) and permits definition of a Hamiltonian energy integral

$$H = \frac{1}{2}\left(y_1^2 + y_2^2\right) + a \operatorname{ch} kx_2 \cos kx_1 + b \cos lx_2, \tag{2.4}$$

where $y_1$, $y_2$ are the canonical momenta corresponding to the coordinates $x_1$, $x_2$.

We will determine the singular points of system (2.3) (equilibrium positions) and establish conditions for their stability by studying the topological structure of the energy surface defined by Hamiltonian (2.4) in their vicinity, using the approach described in [4, 5]. We will consider the equations

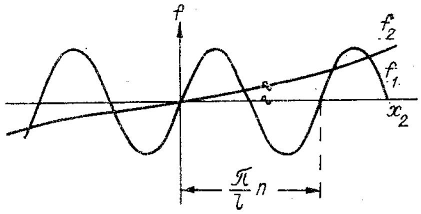


Fig. 2

$$ak \operatorname{ch} kx_2 \sin kx_1 = 0, \quad -ak \operatorname{sh} kx_2 \cos kx_1 + bl \sin lx_2 = 0. \tag{2.5}$$

From the first equation we obtain the solutions $x_1 = \pi i/k$ ($i = 0, \pm 1, \pm 2, \ldots$), with consideration of which the second equation of Eq. (2.5) takes on the form

$$-(-1)^i ak \operatorname{sh} kx_2 + bl \sin lx_2 = 0. \tag{2.6}$$

We will study the solutions of Eq. (2.6) graphically. Thus, for $x_1 = \pi i_e/k$ ($i_e = 0, \pm 2, \ldots$) we write Eq. (2.6) in the form $bl \sin lx_2 = ak \operatorname{sh} kx_2$. Graphing the functions $f_1 = bl \sin lx_2$, $f_2 = ak \operatorname{sh} kx_2$, at the points of their intersection we obtain solutions of Eq. (2.6). It is evident from Fig. 2 that solutions other than $x_2 = 0$ will exist only when the inequality

$$(bl \sin lx_2)'\big|_{x_2=0} > (ak \operatorname{sh} kx_2)'\big|_{x_2=0}$$

is satisfied. The total number of solutions of Eq. (2.6), N, can be determined in the following manner. Let n be the largest number of waves for which the inequality

$$bl > ak \operatorname{sh} k(\pi n/l + \pi/4l)$$

is satisfied. Hence

$$n = [-1/4 + (l/\pi k) \operatorname{arcsh} \sigma], \quad \sigma = bl/ak. \tag{2.7}$$

Then $N = (2n + 2)2 - 1 = 4n + 3$. For $x_1 = \pi i_o/k$ ($i_o = \pm 1, \pm 3, \ldots$) the number $N_1$ of solutions of Eq. (2.6), i.e., equilibrium positions, is the same, or two smaller, i.e., $N_1 = N$ or $N_1 = N - 2$. Thus we have the following discrete set of equilibrium positions:

$$G_{0e} = \{x,\ y;\ y_1 = y_2 = 0;\ x_1 = \pi i_e/k,\ x_2 = 0;\ i_e = 0,\ \pm 2, \ldots\},$$
$$G_{00} = \{x,\ y;\ y_1 = y_2 = 0;\ x_1 = \pi i_0/k,\ x_2 = 0;\ i_0 = \pm 1,\ \pm 3, \ldots\},$$
$$G_{ji} = \{x, y; y_1 = y_2 = 0; x_1 = \pi i/k; x_2 = x_{2j}(\gamma,\ \sigma); i = 0,\ \pm 1, \pm 2, \ldots; j = \pm 1, \pm 2 \ldots\}.$$

We note the significance of the parameters σ and γ in existence of a sequence of equilibrium positions for Eq. (2.3).

We will determine the type of topological structure of the energy surface H = H(x, y) by examining the Gaussian curvature defined by the expression

$$G(x,\ y) = (1 + |\operatorname{grad} H|^2)^{-3/2} \det \operatorname{Hess}(H;\ x,\ y),$$

where

$$\operatorname{Hess}(H; s) = \left(\frac{\partial^2 H(s)}{\partial s\, \partial s}\right), \tag{2.8}$$

(s is the variable system vector, $s = (x_1, x_2, y_1, y_2)$). We will use matrix (2.8) to study the Gaussian curvature.

We will now make use of the following

Definition. The surface H(x, y) is topologically equivalent to the surface H*(x, y) in some related region Γ(x, y) if the following relationship is satisfied:

$$Q^{\mathrm{T}}(x,y) \operatorname{Hess}(H;\ x,\ y) Q(x,\ y) = \operatorname{Hess}(H^*;\ x,\ y), \tag{2.9}$$

where Q(x, y) is some quadratic matrix.

In the case where the region is the vicinity of an equilibrium position, each of which in the given case is a nondegenerate critical point of the surface H(x, y), and the surface H*(x, y) is canonical, i.e., $H^*(x, y)=\sum_{i=1}^{n}(\alpha_i x_i^2+\beta_i y_i^2)$, where $\alpha_i$, $\beta_i$ take on the values $\pm 1$, the given definition is based on Morse's lemma [6].

Since the matrix Hess (H; x, y) is symmetric, a transform (the matrix Q(x, y)) will always exist. However if H*(x, y) is such that the matrix Hess(H; x, y) is diagonal while the region Γ(x, y) is not local, then the question of definition of the transform Q(x, y) remains open.

It is known [4] that if the condition $\Delta_r(x, y) > 0$ for all $i = \overline{1, 2n}$, where $\{\Delta_r\}$ is the sequence of negative main minors of matrix (2.8), is satisfied, then G(x, y) is positively defined. It is then possible to determine a sequence (with consideration of the existence of a sequence of equilibrium positions for Eq. (2.3)) of closed compact invariant regions

$$\bar{\Omega}^+_{i\alpha}(x, y) = \Omega^+_{i\alpha} \cup \partial\Omega^+_{i\alpha},\quad \partial\Omega^+_{i\alpha} \text{ — is the boundary of the region,}$$

$$\Omega^+_{i\alpha} = \Omega^+_i \cap \left\{H(x, y) < \alpha = H^{\partial\Omega^+_i}_{\inf} = \inf_{\partial\Omega^+_i} H\right\},$$

$$\Omega^+_i = \left\{\bigcap_{r=1}^{2n} \Omega^+_{ir}\right\},\quad \Omega^+_{ir} = \{\Delta_{ir} > 0\},\quad i \text{ — is the number of the region,}$$

containing both a stable equilibrium position and periodic (quasiperiodic) system trajectories. In each such region Hamiltonian (2.4) is topologically equivalent to the Hamiltonian

$$H^* = \frac{1}{2}(y_1^2 + y_2^2) + \frac{1}{2}(x_1^2 + x_2^2).$$

We will now consider the cases of indeterminate sign and negative sign determination of G(x, y) (and thus matrix (2.8)).

For the given Hamiltonian (2.4) the matrix (2.8) has the simple form

$$\text{Hess}(H; x) = \begin{pmatrix} \left(\dfrac{\partial^2\Pi(x)}{\partial x_i \partial x_j}\right) & (0) \\ (0) & (1) \end{pmatrix}, \qquad (2.10)$$

where its elements are 2 × 2-matrices. Because of the form of matrix (2.10) it is sufficient to examine its submatrix

$$\left(\frac{\partial^2\Pi(x)}{\partial x_i \partial x_j}\right) = \begin{pmatrix} -ak^2\cos kx_1 \operatorname{ch} kx_2 & -ak^2 \sin kx_1 \operatorname{sh} kx_2 \\ -ak^2 \sin kx_1 \operatorname{sh} kx_2 & ak^2 \cos kx_1 \operatorname{ch} kx_2 - bl^2 \cos lx_2 \end{pmatrix}, \qquad (2.11)$$

which corresponds to study of the topology of the surface Π = Π(x). Using Eq. (2.11), we define

$$\Delta_1(x) = -ak^2 \cos kx_1 \operatorname{ch} kx_2,$$
$$\Delta_2(x) = a^2k^4(-\operatorname{ch}^2 kx_2 + \sin^2 kx_1 + \gamma \cos kx_1 \cos lx_2 \operatorname{ch} kx_2),$$

while

$$\frac{\Delta_2(x)}{\left(1 + \left(\dfrac{\partial\Pi(x)}{\partial x_1}\right)^2 + \left(\dfrac{\partial\Pi(x)}{\partial x_2}\right)^2\right)^{3/2}} = G(x), \qquad (2.12)$$

where G(x) is the Gaussian curvature of the surface Π = Π(x).

For points of the sets $G_{0e}$ and $G_{00}$ we have

$$\Delta_1(M_{0e}) = -ak^2,\quad \Delta_2(M_{0e}) = a^2k^4(-1+\gamma);$$
$$\Delta_2(M_{00}) = ak^2,\quad \Delta_2(M_{00}) = a^2k^4(-1-\gamma).$$

From this it follows that for $\gamma < 1$ all equilibrium positions of the sets $G_{0e}$, $G_{00}$ considered in the subspace X are hyperbolic, and the structure of the surface defined by the potential energy function in the vicinity of these points is equivalent to a saddle. Therefore Hamiltonian (2.4) in the vicinity of the equilibrium position of the sequences $G_{0e}$ and $G_{00}$ is topologically equivalent to Hamiltonian

$$H^* = \frac{1}{2}\left(y_1^2 + y_2^2\right) + \frac{1}{2}\left(\mp x_1^2 \pm x_2^2\right), \tag{2.13}$$

where the upper sign refers to points of the set $G_{0e}$, and the lower sign to $G_{00}$. At $\gamma > 1$ at the equilibrium points of $G_{0e}$ matrix (2.11) is negatively determined. From this it follows that at these points the function $\Pi(x)$ has a maximum, so that Hamiltonian (2.4) in the vicinity of each of these points is topologically equivalent to the Hamiltonian

$$H^* = \frac{1}{2}\left(y_1^2 + y_2^2\right) - \frac{1}{2}\left(x_1^2 + x_2^2\right). \tag{2.14}$$

For all remaining (at $\gamma > 1$) equilibrium positions we obtain

$$\Delta_1(M_{ji}) = -(-1)^i ak^2 \operatorname{ch} kx_2,\quad \Delta_2(M_{ji}) = a^2k^4 \operatorname{ch} kx_2(-\operatorname{ch} kx_2 + (-1)^i\gamma \cos lx_2).$$

Hence at $i = i_0$ it follows that $\Delta_1(M_{ji_0}) > 0$, $\Delta_2(M_{ji_0}) = a^2k^4 \operatorname{ch} kx_2(-\operatorname{ch} kx_2 - \gamma\cos lx_2)$, and for sufficiently large $\gamma$ at $j = j_0$ satisfaction of the inequality $\Delta_2(M_{j_0i_0}) > 0$ is possible. Therefore, aside from maximum points and saddle points, in the vicinity of which Eqs. (2.14), (2.13) are valid, at $i = i_0$, $j = j_0$ there exist points where the function $\Pi(s)$ is at a minimum, thus allowing representation of Hamiltonian (2.4) in their vicinity in the form of Eq. (2.9) so that these are stable equilibrium points of system (2.3).

Thus, the model admits the existence of magnetic traps in a manner similar to the way the limited model three-body problem of celestial mechanics admits existence of gravitational traps corresponding to the libration points $L_4$ and $L_5$ [7]. Here the existence of stable equilibrium positions contradicts Ernshaw's theorem [1] which states the impossibility of a stable configuration created by permanent magnets, and can be explained by the lack of consideration of displacement along the z coordinate, i.e., by the limited formulation of the problem.

Our goal now will be to establish conditions for existence of trajectories corresponding to longitudinal motion (along the axis $Ox_1$) with limited change in the coordinates $y_1$, $x_2$, $y_2$ and to study the stability of such a trajectory with respect to small initial perturbations. We can relate the search for such conditions to establishing the possibility of representing Hamiltonian (2.4) in some tube $T(x, y) = \{x, y\colon |x_1| < \infty,\ |x_2| < \Delta_0,\ \Delta_1 < H(x, y) < \Delta_2,\ \Delta_1 < C = \text{const}\}$ in the equivalent form $H^*(x, y)$, the trajectories of which have the required properties, and then, to establishment of time limits or other conditions for maintenance of these properties.

Consideration of the trajectories of Hamiltonian (2.4) at $\gamma < 1$ on the basis of an equivalent $H^*(x, y)$ of the form of Eq. (2.13) in the tube T shows the impossibility of existence in system (2.3) of trajectories corresponding to the limiting motion. Therefore we will turn to consideration of the case $\gamma > 1$. Here the topological structure of the force field (potential energy) is insured by high parameter values ($\gamma = 62.5$, $\sigma = 25.0$). By Eq. (2.7) the quantities $\gamma$ and $\sigma$ define the value $n = 1$. Below we will show that the condition $\gamma \gg 1$, $n \geqslant 1$ is significant in determining the possibility of reduction to equivalent structures.

By using reductions of the relative character of equilibrium positions it can be established that upon change in the coordinates $x_1$, $y_1$, $x_2$, $y_2$ in the tube

$$T(x, y) = \{x, y : |x_1| < \infty,\ |x_2| < x_2(M_{1i_e}),\ \Pi(M_{1i_e}) < H(x, y) < \Pi(M_{0i_e})\},$$

where $M_{1i_e}$ is the second point in the sequence of equilibrium positions, corresponding to $i_e$, $M_{0i_e}$ is the first point of the same sequence, the maximum point of the function $\Pi(x)$, Hamiltonian (2.4) is topologically equivalent to Hamiltonian

$$H^* = \frac{1}{2}\left(y_1^2 + y_2^2\right) + \frac{1}{2}\left(\cos x_1 + \cos x_2\right). \quad (2.15)$$

Hamiltonian (2.15) can be analyzed easily. From Eq. (2.15) we obtain two independent systems $H_1$ and $H_2$, where

$$H_1 = \frac{1}{2}\left(y_1^2 + \cos x_1\right), \quad H_2 = \frac{1}{2}\left(y_2^2 + \cos x_2\right).$$

Hence, if the values of the energy $H_1$ are such that the initial point $(x_1^0, y_1^0)$ is located beyond the separatrix dividing the region of periodic motions of the system $H_1$ from the region of motions departing to infinity along the axis $Ox_1$ at finite $y_1$, and the values of energy $H_2$ are such that the initial point $(x_2^0, y_2^0)$ lies within the region of periodic motions of the system $H_2$ ($H_2 < H_{2c}$, where $H_{2c}$ is the energy corresponding to motion along the separatrix), the trajectory obtained from the initial point $(x_1^0, y_1^0, x_2^0, y_2^0)$ will correspond to the desired one. Naturally, since the given system (2.3) is bound (which leads to redistribution of the initial energy between subsystems $H_1$ and $H_2$ while maintaining the integral $H = H_1 + H_2$), there must exist a moment of time $t^*$ at which the energy $H_{2c}$ in system $H_2$ will be exceeded. Nevertheless, it can be expected that the interval $[0, t^*]$ will not be brief. Such a possibility may be based on the presence in system (2.3) of an energy localization effect [8], in which not all the energy of one of the mutually coupled systems is transferred to the other. It can be proposed that the possibility of existence of a similar quality in the system of Eq. (2.3) will depend on how much the transform $P(x)$ which changes $\Pi(x)$ to $\Pi^*(x)$ for the parameter range limited by the conditions $\gamma \geqslant 1$, $n \geqslant 1$, differs from an identity.

We will note that for $\gamma$, $\sigma$ values for which $n = 0$ such an approach leads to a different result. Thus we will consider a system with $\gamma = 5.625$, $\sigma = 7.5$ at $a = 0.5$, $k = 4$, $b = 5$, $l = 3$. Such values of $\gamma$ and $\sigma$ insure the presence of only three sequences of equilibrium positions for system (2.3), $G_{0e}$, $G_{00}$ and $G_{1i_e}$

$$G_{1i_e} = \{x, y : y_1 = y_2 = 0;\ x_1 = \pi i_e/k,\ x_2 = \pm x_2(\gamma, \sigma)\}.$$

The equilibrium point sequences $G_{00}$ and $G_{1i_e}$ are saddle points for the surface $\Pi = \Pi(x)$, while at the equilibrium positions $G_{0e}$ the function $\Pi(x)$ has a maximum, i.e., in accordance with Ernshaw's theorem all equilibrium positions are unstable. For the case considered the surface $\Pi = \Pi(x)$ cannot be reduced to the form of Eq. (2.12). The topological equivalent of Hamiltonian (2.4) in the tube $T(x, y)$ will be the Hamiltonian

$$H^* = \frac{1}{2}\left(y_1^2 + y_2^2\right) + \frac{1}{2}\left(-x_2^2 + \cos x_1\right). \quad (2.16)$$

Examination of the trajectories of Hamiltonian (2.16) reveals the absence of trajectories limited in the coordinates $x_2$, $y_2$. Thus, the limitation $n \geqslant 1$, where $n$ is determined from Eq. (2.7), is significant and determines the possibility of representing Hamiltonian (2.4) in the equivalent (within tube T) form of Eq. (2.15).

## LITERATURE CITED


1. V. V. Kozorev, Dynamic Systems of Magnetically Interacting Free Bodies [in Russian], Naukova Dumka, Kiev (1982).
2. Magnetic and Magnetodynamic Supports [in Russian], Énergiya, Moscow (1968).
3. I. S. Gradshtein and I. M. Ryzhik, Tables of Integrals, Sums, Series, and Products [in Russian], 4th edn., revised by Yu. V. Geronimus and M. Yu. Tseitlin, Fizmatgiz, Moscow (1963).
4. A. A. Martynyuk and N. N. Kozhukhovskii, "Stability of Hamiltonian System equilibrium positions," Prikl. Mekh., 16, No. 12 (1980).
5. N. N. Kozhukhovskii, "Instability of Hamiltonian systems with one degree of freedom," Mat. Fiz., No. 29 (1981).
6. M. M. Postnikov, Introduction to Morse Theory [in Russian], Nauka, Moscow (1971).
7. A. P. Markeev, Libration Points in Celestial Mechanics and Cosmodynamics [in Russian], Nauka, Moscow (1978).
8. A. A. Ovchinnikov and N. S. Érikhman, "Localization of oscillatory energy at high excitation levels. Oscillatory excitons," Usp. Fiz. Nauk, 138, No. 2 (1982).